The perfect gas with radiation

- additionally to the gas pressure P_{gas} also the photons in the stellar interior contribute considerably to the total pressure P such that

$$P = P_{\rm gas} + P_{\rm rad} \,.$$

- because radiation is essentially that of a black body, its pressure P_{rad} is given by

$$P_{\rm rad} = \frac{1}{3}U = \frac{a}{3}T^4 \,,$$

where U is the energy density and a is the radiation density constant $a = 7.56464 \times 10^{-15}\,{\rm erg\ cm^{-3}\ K^{-4}}$. The total pressure is then

$$P = P_{\rm gas} + P_{\rm rad} = \frac{\Re}{\mu}\rho T + \frac{a}{3}T^4 \,.$$

- We define

$$\beta := \frac{P_{\rm gas}}{P} \,, \quad 1-\beta = \frac{P_{\rm rad}}{P} \,.$$

For $\beta = 1$ the radiation pressure is zero, while $\beta = 0$ means that the gas pressure is zero.

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$$\beta := \frac{P_{\rm gas}}{P} \,, \quad 1-\beta = \frac{P_{\rm rad}}{P} \,.$$

From differentiating above expression we also get ("β - derivatives")

$$\left(\frac{\partial \beta}{\partial T}\right)_P = -\left[\frac{\partial(1-\beta)}{\partial T}\right]_P = -\frac{4}{T}(1-\beta) \,,$$
$$\left(\frac{\partial \beta}{\partial P}\right)_T = -\left[\frac{\partial(1-\beta)}{\partial P}\right]_T = \frac{1}{P}(1-\beta) \,.$$

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Thermodynamics quantities:

$$\varrho = \frac{\mu}{\Re}\frac{1}{T}\left(P - \frac{a}{3}T^4\right) \,.$$

With the definitions $\alpha := \left(\frac{\partial \ln \varrho}{\partial \ln P}\right)_{T,\mu}$, $\delta := -\left(\frac{\partial \ln \varrho}{\partial \ln T}\right)_{P,\mu}$, $\varphi := \left(\frac{\partial \ln \varrho}{\partial \ln \mu}\right)_{P,T}$

we have $\alpha = \frac{1}{\beta}$, $\delta = \frac{4-3\beta}{\beta}$, $\varphi = 1$.

For mono-atomic gas we have for the internal energy per unit mass

$$u = \frac{3}{2}kT\frac{n}{\varrho} + \frac{aT^4}{\varrho} = \frac{3}{2}\frac{\Re}{\mu}T + \frac{aT^4}{\varrho} = \frac{\Re T}{\mu}\left[\frac{3}{2} + \frac{3(1-\beta)}{\beta}\right] \,.$$

From $c_P := \left(\frac{dq}{dT}\right)_P = \left(\frac{\partial u}{\partial T}\right)_P + P\left(\frac{\partial v}{\partial T}\right)_P$

$$c_P = \left(\frac{\partial u}{\partial T}\right)_P + P\left(\frac{\partial v}{\partial T}\right)_P = \left(\frac{\partial u}{\partial T}\right)_P - \frac{P}{\varrho^2}\left(\frac{\partial \varrho}{\partial T}\right)_P$$ and

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from differentiating the expression of the internal energy u w.r.t. T we obtain with the help of the expressions for the "β - derivatives"

$$\left(\frac{\partial u}{\partial T}\right)_P = \frac{\Re}{\mu}\left[\frac{3}{2} + \frac{3(4+\beta)(1-\beta)}{\beta^2}\right] \,.$$

For the specific heat at constant pressure we get with the previously derived expression(s)

$$c_P = \frac{\Re}{\mu}\left[\frac{3}{2} + \frac{3(4+\beta)(1-\beta)}{\beta^2} + \frac{4-3\beta}{\beta^2}\right] \,,$$

and for adiabatic (dimensionless) temperature gradient (it is a purely TD quantity!)

$$\nabla_{\rm ad} = \frac{\Re\delta}{\beta\mu c_P} = \left(1 + \frac{(1-\beta)(4+\beta)}{\beta^2}\right) / \left(\frac{5}{2} + \frac{4(1-\beta)(4+\beta)}{\beta^2}\right) \,.$$

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$\beta := \frac{P_{\rm gas}}{P}\,,\quad 1-\beta = \frac{P_{\rm rad}}{P}\,.$

The last two expressions become for a monoatomic gas with

$\beta \to 1$: $\quad c_P = 5\Re/(2\mu)\quad$ and $\quad \nabla_{\rm ad} = 2/5,$

$\beta \to 0$: $\quad c_P \to \infty\quad$ and $\quad \nabla_{\rm ad} \to 1/4\,.$

The following expression plays an important role in stellar structure and oscillations

$$\frac{1}{\gamma_{\rm ad}} := \left(\frac{d\ln\varrho}{d\ln P}\right)_{\rm ad} ,$$

which with the help of the differential form of the EOS

$$\frac{d\varrho}{\varrho} = \alpha\frac{dP}{P} - \delta\frac{dT}{T} ,$$

and adiabatic condition $PdT/(TdP) = \nabla_{\rm ad}$ becomes

$$\gamma_{\rm ad} = \frac{1}{\alpha - \delta\nabla_{\rm ad}}\,.$$

Note: every TD quantity depends on 3 other TD quantities (Maxwell relations).

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$$\gamma_{\rm ad} = \frac{1}{\alpha - \delta\nabla_{\rm ad}}\,.$$

For $\beta = 1$: $$\gamma_{\rm ad} = \frac{1}{1-\nabla_{\rm ad}}\,,$$

with $\nabla_{\rm ad} = 0.4 \rightarrow \gamma_{\rm ad} = 5/3$

For $\beta \rightarrow 0$: $$\nabla_{\rm ad} \to \frac{1}{4}\,,\quad \gamma_{\rm ad} \to \frac{4}{3}\,.$$

"adiabatic exponents" introduced by Chandrasekhar

$$\Gamma_1 := \left(\frac{d\ln P}{d\ln\varrho}\right)_{\rm ad} = \gamma_{\rm ad}\,,$$

$$\frac{\Gamma_2}{\Gamma_2 - 1} := \left(\frac{d\ln P}{d\ln T}\right)_{\rm ad} = \frac{1}{\nabla_{\rm ad}}\,,$$

$$\Gamma_3 := \left(\frac{d\ln T}{d\ln\varrho}\right)_{\rm ad} + 1\,,$$

and obey the relation

$$\frac{\Gamma_1}{\Gamma_3 - 1} = \frac{\Gamma_2}{\Gamma_2 - 1}\,.$$