Resulting solutions for a linear set of models for fixed $M$ and varying $M_c$, i.e. varying $q_0 = M_c/M$ (quasi-evolution): no degeneracy

- strong degeneracy $M_c$ due to H shell burning (quasi-evolution with time)

Integration of (isothermal) core and envelope models in (complete = thermal and hydrostatic) equilibrium

- $\epsilon_n = X_c$, $P_c = P_L$, $T_c = T_L$,
- $L_c = 0 = 0$

Calculations (simple quasi-evolution) assumed complete equilibrium:

- hydrostatic equ.: $\frac{\partial P}{\partial m} = -\frac{GM}{4\pi r^4}$
- thermal equ.: $\frac{\partial T}{\partial m} = 0$

"sudden" jump due to core compression ($\partial Q/\partial m$)

Evolution through helium burning: crossing the Hertzsprung gap

$M = 3M_\odot$

$M_c (R_c)$ due to H shell burning

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Evolution through helium burning: intermediate-mass stars (>2.5 $M_\odot$)

Full numerical computations ($X=0.70$, $Y=0.28$, $Z=0.02$) for $M = 5M_\odot$

A-C: central H burning exhausted after ~79 mill.y; convection also stops. At ~ same time H shell burning starts, initially in rather broad shell, where $X_c$ increases with radius, left by shrinking convective core:

BUT if stellar structure changes occur on a time scale $<<$ than nuclear time scale we have also to consider:

- $\frac{\partial P}{\partial m} = -\frac{GM}{4\pi r^4}$

- $\frac{\partial T}{\partial m} = \epsilon_n - \epsilon_g$

- $\epsilon_g := \gamma \frac{\partial P}{\partial T} + \frac{\partial P}{\partial \rho} \frac{\partial \rho}{\partial T}$

- Gravitational energy released from contracting core

- Time scale $t_{KH}$

- Typically Kelvin-Helmholtz

C-D: after C evolution accelerates $\sim$ abscissa scale; model no longer in thermal eq. i.e. $|\epsilon_g| > 0$

- core shrinks below and envelope expands above H shell source $\rightarrow$ radial motion of mass elements
Evolution through helium burning: intermediate-mass stars (>2.5 $M_\odot$)

Full numerical computations ($X=0.70$, $Y=0.28$, $Z=0.02$) for $M = 5 M_\odot$.

**C-D:**
- radial motion: core contraction & envelope expansion around H shell
- $\alpha = 0.86$ (mirror principle).

- Such massive star starts at MS with relatively low central density, core remains non-degenerate during present contraction; this leads to heating, because of

  If helium is ignited in non-degenerate core, rapid contraction stops. Core contraction (C-D) time ~ $t_{KH}$ (~3 million years).

- Envelope expansion transforms the star into a red giant very rapidly ($t_{KH}$), little chance to observe massive stars during this short phase of evolution:
  - explains Hertzsprung gap

**D-G:**
- central He burning
- D-E: red-giant region in H-R diagram; close to Hayashi line; very deep surface CZ down to $m/M = 0.17$ (deeper with $M$).
- Reddish layers, where $\mu$ has been modified (dotted areas) by earlier H burning; products (heavier elements such as those from the CNO cycle) are mixed and transported to surface.

- 1st dredge up.
- At first dominant reaction is triple-$\alpha$: $\nu = 3 ^{14}$N $\rightarrow ^{17}$O. Later, $^{15}$N $\rightarrow ^{18}$F takes over.
- With $^{15}$N becoming rare, depletion of $^{12}$C: account of $^{12}$C is larger than production of $^{12}$C, i.e. $^{12}$C decreases again after maximum.

- Central He burning lasts ~ 16 My, which is ~ 20% of H-MS phase; this is rather long because most of $L$ is from H burning shell ($\nu = 7\%$ at 16 My and $42\%$ at $G$).
- Core radius $R_{c_{He}}$ changes rather little with time for $Y > 0.3$, but is affected by convective overshooting (shrinkage).
Evolution through helium burning: intermediate-mass stars (>2.5 M$_\odot$)

Full numerical computations (X=0.70, Y=0.28, Z=0.02) for M = 5 M$_\odot$

D-G: central He burning

E-F-G: after E star moves first down Hayashi line, and then to the left (blue). The “bluest” point F is reached after ~14 mill y (~88% of central He-burning phase); the track leads back to G. These loops cross the classical instability strip (IS) for stars with M>5M$_\odot$, where δ Cepheids (pulsating stars) are observed.

The Cepheid Phase: (C)-E-G

typical giant stars located within the IS, which is only about 600-1000 K in width. Pulsate with large amplitude in typically one (radial) pulsation mode (breathing mode). Pulsations are driven by Carnot-like heat engine in the He ionization layers (κ mechanism).

in our case stellar model crosses IS 3 times:
C-D (too fast), (E-F), (F-G).

theory predicts for pulsation period $\Pi$:

$\Pi \propto (L)^{-1/2} \propto R^{1/2}/M^{1/2} \propto \Omega_{\text{Edd}}$

E-F: $R_\Pi \rightarrow \Pi_{\text{E}}$
F-G: $R_\Pi \rightarrow \Pi_{\text{F}}$

for $L_{\text{E}} \rightarrow R_{\text{E}} \rightarrow \Pi_{\text{E}} \rightarrow \Pi_{\text{F}} \rightarrow L_{\text{F}}$

important for distant measurements
Evolution through helium burning: intermediate-mass stars (>2.5 $M_\odot$)

To loop or not to loop

(E-G)

Evolution through loops (for $M \leq 7 M_\odot$) rather slow → use models in complete equilibrium.

Consider separate models for He core ($M_c, R_c, I_0$) and H-rich envelope; H-shell burning provides additional $L, T_{\text{eff}}, P_0, T_0$.

Envelope:
- Inner BC: $R_c, I_0 = 0$ @ $m = M_c$
- Solution: $L, T_{\text{eff}}, P_0$
- Need simple non-monotonic function ($R_c, M_c, \Delta m, \Delta X_H$) for distance measure from Hayashi line.

Core:
- Effective (core) surface potential $\varphi := h(\Delta m, \Delta X_H) \frac{M_c}{R_c}$
- From numerical calculations:
  - For $h = 1$ & constant $M_c$, move upwards.
  - Five sequences of envelope solutions for $h > 1$ & with constant $M_c$ but varying $R_c$, move upwards.

Effective surface potential $\varphi := h(\Delta m, \Delta X_H) \frac{M_c}{R_c}$ is indeed indicator of distance from Hayashi line.

For $\varphi > \varphi_{\text{CTR}}$, all envelope solutions move upwards.

From numerical calculations:
- $h = e^{\text{constant} \cdot \Delta m \cdot \Delta X_H}$

Five sequences of envelope solutions for $h = 5/3M_\odot$

To loop or not to loop

(E-G)

$M = 5M_\odot$

1. $h = 1$ for ($\Delta m = 0, \Delta X_H = 0$) step function
2. $\varphi$ is indeed indicator of distance from Hayashi line.
3. For $\varphi > \varphi_{\text{CTR}}$, all envelope solutions move upwards.
4. From numerical calculations:
   - $h = e^{\text{constant} \cdot \Delta m \cdot \Delta X_H}$

Both effect, the increase of $M_c$ (H-shell burning outwards) and decrease of $R_c$ after central He burning ($Y < 0.1$): rapid decrease of $R_c$.

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Evolution through helium burning: intermediate-mass stars (>2.5 M\(_\odot\))

To loop or not to loop (E-G)

\(\varphi_{\text{crit}}\) depends on:
- timescales of H-shell and central He-burning phase.
- convection and in particular convective overshooting, which affects \(M_c\) (increases convective core), modifies \(\varphi_{\text{crit}}\).

![Graph showing evolution through helium burning](image)

- Mirror principle: core contracts, He region between shell sources expands, envelope contracts.
- \(T\) drops in outward-moving H shell: H-burning stops: core contracts, env. expands around He-burning shell.
- \(L\) increases rapidly with increasing C-O core mass.
- Outer convective envelope reaches down and penetrates into region where H-burning shell had produced heavier elements (CNO) and dredge up.
- Inward motion of CZ reaches \(T\sim3\times10^8K\) H reignites.