Evolution of the C-O core

Further evolution details depend on whether C-O core becomes degenerate or not in the ensuing contraction phase.

Estimating critical core mass $M_{\text{crit}}$, which determines whether contraction will increase $T_0$ or if the core becomes degenerate:

- consider (approximate) EOS interpolating between both (non-deg. – deg.) regimes:

$$P = \frac{\rho G M}{R} \left( \frac{\rho}{\rho_0} \right)^{\Gamma}$$

- rough estimate of central $T_0$ values from hydrostatic equation

$$P_0 = \frac{GM_0^2}{R_0} = \frac{G M_0^2}{R_0} \left( \frac{\rho}{\rho_0} \right)^{\Gamma}$$

use EOS for $P_0$

- dominated by non-deg. $\mu_e$ ($\approx 2/3$, $\mu_0 \geq 12$) $\frac{\rho}{\rho_0} \gg 10^6 \text{g cm}^{-3}$

- dominated by deg. $\mu_e$ ($\approx 2$) $\frac{\rho}{\rho_0} \ll 10^6 \text{g cm}^{-3}$

REL. NON-REL.

$10^{15} - 10^{13}$

- rough estimate of $T_0$ values from hydrostatic equation

$$P_0 = \frac{GM_0^2}{R_0} = \frac{G M_0^2}{R_0} \left( \frac{\rho}{\rho_0} \right)^{\Gamma}$$

this shows that $T_0$ increases with $\rho_0$.

Collapse and final explosions

Stars with initial mass of less than ~9 $M_\odot$ (this limit depends strongly on mass loss) develop degenerate cores and if shell sources cannot increase $M_c$ to $M_{\text{ch}}$, the star becomes a WD.

Other stars undergo core collapse (such as those with neutron stars as remnants) or explosions, thereby ejecting a large part of their mass (supernova).

Evolution of the C-O core

Homologous contraction of a gas sphere

tracks for different $M_c$ values

- non-degenerate region: $T_0 \approx T_0^\text{max} \approx 10^{6}$
- for low $\rho_0$ and small $M_c$, the temperature $T_0$ increases up to a maximum value $T_0^\text{max}$ after which it decreases until $T_0 \approx 6$ (A, B, M, M in Ch. 28).

- with non-relativistic degeneracy becomes important, we rewrite EOS as $\Gamma = \frac{4}{3} + \chi$ with $\chi \approx 0$ for $\rho/\rho_0 \gg 10^{6}$ (g cm$^{-3}$):

$$P_0 = \frac{GM_0^2}{R_0} = \frac{G M_0^2}{R_0} \left( \frac{\rho}{\rho_0} \right)^{\Gamma}$$

- for $M_0 > M_{\text{ch}}$: $T_0$ continuously increases with $\rho_0$.

$\rho_0$ dominated by non-deg. $\mu_e$ ($\approx 2$) $\frac{\rho}{\rho_0} \gg 10^6 \text{g cm}^{-3}$

- for $M_0 \ll M_{\text{ch}}$: $T_0 \approx \frac{2}{3} \frac{M_0}{T^2}$ and decreases afterwards

$\rho_0$ dominated by deg. e- ($\approx 2$) $\frac{\rho}{\rho_0} \ll 10^6 \text{g cm}^{-3}$

$\rho_0$ increases with $M_c$ C-burning starts, e.g. by pycnonuclear reactions, in degenerate core C-burning starts (explosive: type 1.5 SN). This is the case for stars with typical masses $4.5 < M/M_\odot < 8$.

- pair creation

- C burning in non-degenerate core, but crosses later region of pair creation (photon energy so large that it can create $\text{ee}^-$ pair) $\Rightarrow M_c < 4.5 < \text{core collapse until } T_0 \text{ so high that O burning starts and core collapse would stop}$.

- core collapse (may lead to neutron-star formation and ejection of envelope) $\Rightarrow$ type II supernova.

For cores with $M_c > M_{\text{ch}}$: $T_0 \approx 0.5 \times 10^9$ K.
Collapse and final explosions

The carbon flash (B*):
- For M > 8M⊙ shell-burning source increased core mass nearly to Mch, thereby also increasing ρ0, released energy of core contraction is transported by e- conduction towards centre, where T_{max} > T_{core} because of neutrino losses, which carry outwards the energy. Once ρ is high enough C burning ignites.
- C ignition either in centre or shell of T_{max} core stability crucially depends on neutrino losses if ρ_{c} < ρ_{t} → core becomes unstable → (violent) C flash
- core stability crucially depends on neutrino losses ρ_{0}:

How violent is C flash:
for a mixture of equal C & O, C burning releases ~ 2.5x10^{17} erg/g and O twice as much. If all this energy is used to heat the core, it can reach temperatures as indicated by the dashed curve, labelled "C,O". At this high temperatures, T ≈ 10^{10} K we have photodisintegration.

Collapse and final explosions

Nuclear statistical equilibrium (in a plasma of photodisintegration)

@ Tabout 10^{10} K photons γ so energetic as to cause photodisintegration in the nuclei in the gas (α decay), e.g.,

\[ ^{20}\text{Ne} + \gamma \rightarrow ^{16}\text{O} + \alpha \]

Processes of disintegration (dissociation) and inverse reaction similar to ionization and recombination of atoms, statistical equilibrium can be described via SAHA equation, e.g.

\[ n_{0}n_{a} = \left( \frac{2\pi m_{a}m_{b}kT}{\pi m_{b}} \right)^{3/2} \frac{G_{0}G_{a}}{G_{ba}} e^{-Q/\Delta T} \]

where G_{0}, G_{a}, and G_{ba} are the statistical weights, while Q is the difference of binding energies

\[ Q = (m_{O} + m_{α} - m_{Ne})^{2} \]

Difference of binding energies

\[ \Delta Q = (13m_{α} + 4m_{Ne} - m_{Fe})^{2} \]

Nuclear statistical equilibrium defines ratio \( n_{0}/n_{a} \)

\[ \gamma + ^{56}\text{Fe} \rightarrow 13\alpha + 4\alpha \quad \text{(13α)} \]

\[ \gamma + ^{56}\text{Fe} \rightarrow ^{50}\text{Fe} + ^{24}\text{Mg} \quad \text{(4k)} \]

Note: for Fe:
\[ n_{0}/n_{a} = 13/15 \]
\[ n_{0}/n_{a} = 4/13 \]

\[ n_{0}/n_{a} = 2 \text{ equ. (1)+(2) for 2 unknowns } m_{α} \text{ and } n_{0} \]
Collapse and final explosions
Nuclear statistical equilibrium (in a plasma of photodisintegration)

@ Tab about $10^{10}$ photons/y, so energetic as to create $\nu-\bar{\nu}$ pairs = photodisintegration, e.g.

for given ratio, e.g. $m_\odot/m_\odot = 4/13$ (56Fe), and given $\rho$ & $T$, nuclear equilibrium demands:

nr. of protons

nr. of neutrons

In nuclear statistical equilibrium at moderate $T$ one expects nuclei of the iron group, which with increasing $T$ disintegrate into $\alpha$ particles, protons and neutrons.

Collapse and final explosions
Hydrostatic and convective adjustment during C flash

- during He flash star stays very close in hydrostatic equilibrium: convection removes energy.
- for C flash the situation is very different:
  - in a single thermal run away, after the C flash ($T_\odot$), $T$ so fast that additional reactions (O burning) take place: core regions are then so hot that statistical equilibrium between Fe & He is reached: degeneracy is removed, pressure increases -> central regions expand.

- time scale $\tau_C$ determined by change of $T$ & internal energy ($T/\dot{E}_{\text{int}})$:

other (outer) core regions react on the hydrostatic time scale:

$\tau_h \approx \frac{\rho_{\odot}}{T_{\odot}} \frac{\dot{E}_{\text{int}}}{T_{\odot}}$

- if $\xi = \tau_C/\tau_h > 1$ core follows expansion quasi-hydrostatically
- if $\xi < 1$ outer layers can not expand rapidly enough, compression wave will move outwards with $c_s$ (outwards travelling shock wave)

convective in core will (a) transport part of surplus energy to outer layers:

(b) bring new fuel into burning layers:

if $\xi = \tau_C/\tau_h > 1$ convection carries all energy away from core

Combinations of $\tau_C$ & $\tau_h$:

- $\xi = 1$ compression wave will stop outwards
- $\xi > 1$ if compression wave does not ignite fuel, then $T_{\odot}$ reaches $T_{\text{ign}}$ due to energy transport (convection or conduction)

- $\xi < 1$ if compression wave does ignite fuel, then it propagates outwards

2 different types of combustion front:

(a) matter in front enters discontinuity of compression wave (shock wave) with supersonic velocity and is compressed & heated: if matter is ignited combustion front coincides with shock front -> detonation front

(b) if compression in shock wave does not ignite the fuel, then $T_{\odot}$ reaches $T_{\text{ign}}$ due to energy transport (convection or conduction)

Two different types of combustion front:

- matter in front enters discontinuity of compression wave (shock wave) with supersonic velocity and is compressed & heated: if matter is ignited combustion front coincides with shock front -> detonation front

- if compression in shock wave does not ignite the fuel, then $T_{\odot}$ reaches $T_{\text{ign}}$ due to energy transport (convection or conduction) -> slower, subsonic motion of the burning front with a discontinuity in pressure and density drop (inwards)

Combustion fronts (during C flash)

$\tau_C \approx 10^{-6}$ s: at the onset is rather short -> burning proceeds at such high rates that fuel in mass shell is consumed essentially instantly & layer above has not time to adjust.

Layer ahead is heated to ignition either by compression or energy transport, and flash proceeds outwards (burning confined to very thin layer) -> outward moving combustion front.

In both cases deviation from hydrostatic equilibrium mainly confined in thin shell of $P, \rho$ discontinuity, and energy release.
Collapse and final explosions

Collapse of cores of massive stars (C,D): $M_{ch} < M < 40M_\odot$

core "misses" non-relativistic degeneracy and heats up during contraction until ignition of next heavier element. Core is then either non-degenerate (large $M$) or degenerate, but still in a region "above" $\alpha=4/3$, i.e. gravothermal heat capacity $c^*<0$ → burning is stable!

after several cycles of nuclear burning and contraction, core will heat up to Si burning; burning in several shell sources produces layers of different chemical elements → onion-shell model.

Finally central regions of core reach $T$ at which abundances are determined by nuclear statistical equilibrium.

$n\uparrow M_\odot \lesssim 4M_\odot$

$n\uparrow M_\odot \lesssim 4M_\odot$

In summary:

-the pressure and energy loss due to electron capture accelerates the contraction (collapse).

Infall of matter (massive stars: C,D)

Because of infall $\rho$ approaches that of neutron stars ($\rho \approx 10^{14}$ g cm$^{-3}$). Matter becomes incompressible (EOS is "stiff").

Complete elastic reflection would bring whole collapse only to original state before the collapse; we need extra energy to expel the envelope.

Remnant (neutron star) is somewhat compressed by inertia beyond equilibrium state and afterwards, acting like a "spring", it expands and pushes back the infalling matter.

This creates a pressure wave, steepening if it enters regions of lower density.

However, a substantial fraction of the energy in this rebounding pressure wave will be used up to disintegrate remaining Fe (in envelope) into free nucleons, i.e. only a small fraction of the (rebounced) kinetic energy remains in the shock wave for lifting the envelope (also major energy loss due to neutrinos → only 1% of initial kinetic energy available for lifting envelope).

During (core) collapse, neutrino production by neutronization becomes dominant. Because of large density, matter becomes opaque to neutrinos, i.e. free-mean path is reduced and so the neutrino velocity $[v > 3 \times 10^{10}$ g cm$^{-3}$ neutrinos are trapped, because their velocities are smaller than the infall velocity (free-fall)]. Influence further neutronization, i.e. neutronization stops @ $\rho \sim 3 \times 10^{14}$ g cm$^{-3}$ @ equilibrium. Collapse stops @ $\rho > 10^{15}$ g cm$^{-3}$.
**Collapse and final explosions**

**Neutrinos in MeV range have mean free path \( \langle \mu = 1 \rangle \)**

\[
\lambda_\nu = \frac{1}{\sigma_{\nu r}} = \frac{\mu m_\nu}{c^2 v} = 2 \times 10^{19} \text{cm} = 1.3 \times 10^{12} \text{gcm}^{-2}.
\]

**Normal stellar matter**

\( \rho \approx 10^7 \text{gcm}^{-3} \) : \( \langle \mu \rangle \approx 100 \text{ parses} \)

\( \rho \approx 10^8 \text{gcm}^{-3} \) : \( \langle \mu \rangle \approx 10 \text{ cm}^{-2} \)

**Collapsing stellar core**

\( \rho \approx 10^{13} \text{gcm}^{-3} \) : \( \langle \mu \rangle \approx 20 \text{ km} \)

---

**Shock is formed at sphere labelled “Core shock”, within which matter is almost at rest, after sudden stop of core collapse, once density has reached nuclear matter density (\( \rho \approx 10^{14} \text{gcm}^{-3} \)).**

Neutrinos interact with electrons (inelastic scattering) and neutrinos are thermalized, which brings also the weak interaction into equilibrium (already at \( \rho \approx 10^{12} \text{gcm}^{-3} \)) - homologous core.

Sound speed in core is larger than infall velocity. Where both velocities are the same = core shock boundary of homologous core. Thus the sudden stop of collapse causes shock wave at the surface of the homologous core with \( R \approx 30 \text{km} \).

**Reflection of infall (massive stars: C,D) : Supernova explosion**

Shock wave travels outwards through rest of collapsing iron core with energy of about \( 10^{52} \text{erg} \). Matter through which shock travels will be dissociated.

**Supernova simulation**

Figure shows radial motion of various layers and locations of travelling shock. Shock shock at \( R \approx 500 \text{km} \), but gets revived again after about 0.5s, finally exploding the star.

Core (newly-born neutron star) cools rapidly via neutrino-antineutrino pair creation:

\[ \gamma + \gamma \rightarrow e^- + e^+ \rightarrow \nu_e + \bar{\nu}_e , \]

leading to neutrino absorption by nucleons (p,n), thereby transferring energy (heated within \(< 0.3 \text{ s} \)) to the matter which has previously been dissociated by the shock.

Material heated to energy values sufficient to overcome gravitational potential and can therefore be expelled from the star. This mechanism through neutrino heating is called "delayed supernova" mechanism. Time-dependent convection important for explosion mechanism!

Explosion expels matter outside "mass cut" \( \approx 1.6M_\odot \) into ISM. Partially degenerate remnant is new neutron star consisting, after cooling, mainly of fully-degenerate neutrons.

**Some properties of Supernovae (SN)**

SN are amongst the brightest objects in the universe and can be brighter than a whole galaxy for weeks.

- Energy of visible explosion: \( \approx 10^{51} \text{erg} \) (\( \approx 1 \text{ foe} \))
- Total energy: \( \approx 10^{52} \text{erg} \) (most in neutrinos)
- Luminosity: \( \approx 10^{50} \text{L}_\odot \)

SN events are rather rare: some 1 – 10 per century and galaxy. (In our Galaxy only a few have been recorded, the last one in the 17th century: Kepler’s SN: type Ia).

**Classifications of Supernovae (SN)**

**Observational:**

- Type I: no H lines (depending on other spectral features: Ia, Ib, Ic, …)
- Type II: hydrogen lines

**SN progenitor**

- Type I: 2 possibilities
  - Ia: white dwarf accreting matter from (massive) companion in binary system
  - Ib,c: collapse of Fe core in star that blew its H (or He) envelope into space before the explosion
- Type II: collapse of Fe core in normal massive stars (8 – 30 \( M_\odot \))
- Electron-capture SN (Crab nebula?)
Classifications of Supernovae (SN)

Observational:

- Type Ia: White dwarf mass, stripped nova
- Type Ib: White dwarf mass, stripped nova
- Type II: Core collapse
- Type Ib/c: Mixing of core collapse
- Other types

Explosion Energy sources:

- H-recombination
- Radiative decay: $^{56}\text{Ni}$, $^{56}\text{Co}$, $^{56}\text{Fe}$

Supernova explosion

SN 1998bu
Supernova 1994D in NGC 4526

Supernova 2001cm in NGC 5965

Crab nebula in Taurus
(believed to be electron capture type SN)