Chapter 10

Excitation and damping of the oscillations

So far I have almost exclusively considered adiabatic oscillations, and therefore have been unable to investigate the stability or instability of the modes. Such questions are of obvious interest, however. Here I consider some simple aspects of mode excitation, including properties of the nonadiabatic problem. A major goal is to get a feel for the conditions under which a mode may be *self-excited*, *i.e.*, with a positive growth rate. Also, in cases where all modes are damped, they may still be driven to observable amplitudes by stochastic forcing from near-surface convection; this seems, for example, to be the case for solar oscillations.

10.1 A perturbation expression for the damping rate

In the present section I derive an expression which allows an estimate of the growth or damping rate on the basis of the adiabatic eigenfunction. The procedure is to use the perturbation expression in equation (5.73) which was derived from the oscillation equations written as a linear eigenvalue problem in equation (5.56). Now, however, I take the perturbation $\delta\mathcal{F}$ to be the departure from adiabatic oscillations in the momentum equation. From the perturbed energy equation, equations (3.47) and (3.48) it follows that

$$\begin{aligned}\frac{p'}{p} &= \Gamma_1\frac{\rho'}{\rho} + \xi_r\left(\frac{\mathrm{d}\ln p}{\mathrm{d}r} - \Gamma_1\frac{\mathrm{d}\ln\rho}{\mathrm{d}r}\right) + \frac{i}{\omega}\frac{\Gamma_3-1}{p}(\rho\epsilon - \mathrm{div}\,\mathbf{F})' \\ &= \frac{p'_{\rm ad}}{p} + \frac{i}{\omega}\frac{\Gamma_3-1}{p}(\rho\epsilon - \mathrm{div}\,\mathbf{F})' \,, \end{aligned} \qquad (10.1)$$

where I dropped the subscript "0" on equilibrium quantities, and assumed a time dependence as $\exp(-i\omega t)$. Here

$$p'_{\rm ad} = p\,\Gamma_1\frac{\rho'}{\rho} + \xi_r p\left(\frac{\mathrm{d}\ln p}{\mathrm{d}r} - \Gamma_1\frac{\mathrm{d}\ln\rho}{\mathrm{d}r}\right) \qquad (10.2)$$

is the pressure perturbation corresponding to adiabatic oscillations. It follows that the perturbed momentum equation (3.43) can be written, after separation of the time dependence, as

$$-\rho\omega^2\boldsymbol{\delta}\mathbf{r} = -\nabla p'_{\rm ad} + \rho\,\mathbf{g}' + \rho'\mathbf{g} - \frac{i}{\omega}\nabla[(\Gamma_3-1)(\rho\epsilon - \mathrm{div}\,\mathbf{F})'] \,. \qquad (10.3)$$

This is of the form considered in equation (5.56):

$$\omega^2 \boldsymbol{\delta}\mathbf{r} = \mathcal{F}_{\mathrm{ad}}(\boldsymbol{\delta}\mathbf{r}) + \delta\mathcal{F}(\boldsymbol{\delta}\mathbf{r}) \; , \tag{10.4}$$

with

$$\mathcal{F}_{\mathrm{ad}}(\boldsymbol{\delta}\mathbf{r}) = \frac{1}{\rho}\nabla p'_{\mathrm{ad}} - \mathbf{g}' - \frac{\rho'}{\rho}\mathbf{g} \; , \tag{10.5}$$

and

$$\delta\mathcal{F}(\boldsymbol{\delta}\mathbf{r}) = \frac{i}{\omega\rho}\nabla[(\Gamma_3 - 1)(\rho\epsilon - \operatorname{div}\mathbf{F})'] \; . \tag{10.6}$$

As argued in Section 5.5, $\mathcal{F}_{\mathrm{ad}}$ is in fact a linear operator on $\boldsymbol{\delta}\mathbf{r}$. It may be shown that the same is true for $\delta\mathcal{F}$.

Exercise 10.1:

> Show that $\delta\mathcal{F}$ may be obtained as a linear operator on $\boldsymbol{\delta}\mathbf{r}$, assuming the diffusion approximation, equation (3.22). Note that since $\delta\mathcal{F}$ is assumed to be a small perturbation, it may be derived assuming that $\delta\rho$, δp and δT are related adiabatically.

The effects on the frequency of departures from adiabaticity can now immediately be obtained from the perturbation expression (5.73) as

$$\delta\omega^2 = \frac{i}{\omega}\frac{\int_V \boldsymbol{\delta}\mathbf{r}^* \cdot \nabla[(\Gamma_3 - 1)(\rho\epsilon - \operatorname{div}\mathbf{F})']\mathrm{d}V}{\int_V \rho|\boldsymbol{\delta}\mathbf{r}|^2\mathrm{d}V} \; . \tag{10.7}$$

The integral in the numerator can be rewritten as

$$\int_V \operatorname{div}[\boldsymbol{\delta}\mathbf{r}^*(\Gamma_3 - 1)(\rho\epsilon - \operatorname{div}\mathbf{F})']\mathrm{d}V - \int_V \operatorname{div}(\boldsymbol{\delta}\mathbf{r}^*)(\Gamma_3 - 1)(\rho\epsilon - \operatorname{div}\mathbf{F})'\mathrm{d}V \; ; \tag{10.8}$$

the first integral can be transformed, by using Gauss's theorem (3.3), into an integral over the stellar surface which can be neglected, whereas in the second integral we use the continuity equation (3.42). The result is, finally, that the frequency change caused by non-adiabaticity is

$$\delta\omega = \frac{i}{2\omega^2}\frac{\int_V \frac{\delta\rho^*}{\rho}(\Gamma_3 - 1)(\rho\epsilon - \operatorname{div}\mathbf{F})'\mathrm{d}V}{\int_V \rho|\boldsymbol{\delta}\mathbf{r}|^2\mathrm{d}V} \; . \tag{10.9}$$

This is the desired expression. It should be noted that this expression is valid also in the full nonadiabatic case, if the nonadiabatic eigenfunctions are used.

10.1.1 The quasi-adiabatic approximation

To evaluate the integral in the numerator in equation (10.9) we need an expression for $(\rho\epsilon - \operatorname{div}\mathbf{F})'$. Using that $\epsilon = \epsilon(\rho, T)$ (I neglect a possible Eulerian perturbation in the composition) it is easy to see that

$$(\rho\epsilon)' = \rho\epsilon\left[\epsilon_T\frac{T'}{T} + (\epsilon_\rho + 1)\frac{\rho'}{\rho}\right] \; , \tag{10.10}$$

where

$$\epsilon_T = \left(\frac{\partial \ln \epsilon}{\partial \ln T}\right)_\rho , \qquad \epsilon_\rho = \left(\frac{\partial \ln \epsilon}{\partial \ln \rho}\right)_T . \tag{10.11}$$

Similarly, the perturbation in the flux can be evaluated from the diffusion approximation, equation (3.22), and in particular assuming that there are no other contributions (such as convection) to the heat transport. The result is

$$\mathbf{F}' = \left[(3 - \kappa_T)\frac{T'}{T} - (1 + \kappa_\rho)\frac{\rho'}{\rho}\right] F_\mathrm{r}\mathbf{a}_r - \frac{4a\tilde{c}T^3}{3\kappa\rho}\nabla T' , \tag{10.12}$$

where

$$\kappa_T = \left(\frac{\partial \ln \kappa}{\partial \ln T}\right)_\rho , \qquad \kappa_\rho = \left(\frac{\partial \ln \kappa}{\partial \ln \rho}\right)_T , \tag{10.13}$$

and F_r is the equilibrium radiative flux (which is of course in the radial direction). The underlying assumption in the perturbation treatment leading to equation (5.73) and hence (10.9) is that $\delta\mathcal{F}$ should be evaluated for the *adiabatic* eigenfunction. Regardless of the assumption of adiabaticity we may obtain ρ' from the equation of continuity as

$$\frac{\delta\rho}{\rho} = -\mathrm{div}\,(\boldsymbol{\delta}\mathbf{r}), \qquad \rho' = \delta\rho - \xi_r\frac{\mathrm{d}\rho}{\mathrm{d}r} . \tag{10.14}$$

From adiabaticity it follows that the temperature perturbation can be computed from

$$\frac{\delta T}{T} = (\Gamma_3 - 1)\frac{\delta\rho}{\rho}, \qquad T' = \delta T - \xi_r\frac{\mathrm{d}T}{\mathrm{d}r} . \tag{10.15}$$

Hence, given $\boldsymbol{\delta}\mathbf{r}$, ρ' and T' can be computed, and then $(\rho\epsilon - \mathrm{div}\,\mathbf{F})'$ can be obtained from equations (10.10) and (10.12). Since this approximation to the damping rate can be obtained from the adiabatic eigenfunction, it is known as the *quasi-adiabatic* approximation. As the adiabatic eigenfunctions may be chosen to be real, the integrals in equation (10.9) are real, and hence $\delta\omega$ is purely imaginary. Thus it represents a pure damping or excitation, with no effect on the (real) oscillation frequency.

It should be noted that the approximation is not without problems. The perturbation approach is based on the assumption that the perturbation is small. This is true in most of the star, but not very near the surface where nonadiabatic effects become strong. Here nonadiabaticity has a substantial effect on the eigenfunction, and hence an evaluation of the integral in equation (10.9) based on the adiabatic eigenfunctions is questionable. Nonetheless, we may hope that the quasi-adiabatic approximation at least gives an indication of the stability properties of the mode. A separate problem, which would equally affect a full nonadiabatic treatment, is the neglect of convective contributions to the heat flux. This introduces a major uncertainty in the calculation of the stability of modes in cool stars with extensive outer convection zones. Indeed, it was shown by Baker & Gough (1979) that the transition to stability at the cool side of the Cepheid instability strip most likely is the result of the increased importance of convection. Houdek (2000) made a more detailed analysis, based on a sophisticated mixing-length model of the interaction between convection and pulsations, and similarly showed that convection caused the return to stability at the cool side of the instability region for δ Scuti variables.

10.1.2 A simple example: perturbations in the energy generation rate

To illustrate some simple properties of equation (10.9) I consider the case where the nonadiabaticity is dominated by the energy generation. Here it is convenient to work purely in terms of Lagrangian perturbations, by noting that

$$(\rho\epsilon - \operatorname{div}\mathbf{F})' = \delta(\rho\epsilon - \operatorname{div}\mathbf{F}) \,, \tag{10.16}$$

since the equilibrium model is assumed to be in thermal equilibrium. Also, it is obvious that $\delta(\rho\epsilon)$ can be obtained from an expression analogous to equation (10.10). Neglecting the term in $\mathbf{F}$ and using equation (10.15) we find

$$\delta\omega = \frac{i}{2\omega^2} \frac{\int_V \left|\frac{\delta\rho}{\rho}\right|^2 (\Gamma_3 - 1)[\epsilon_\rho + 1 + (\Gamma_3 - 1)\epsilon_T]\rho\epsilon \mathrm{d}V}{\int_V \rho|\boldsymbol{\delta}\mathbf{r}|^2 \mathrm{d}V} \,. \tag{10.17}$$

Since ϵ_T and ϵ_ρ are positive, and $\Gamma_3 \simeq 5/3$, it is obvious that the integrals in equation (10.17) are positive. With the assumed time dependence as $\exp(-i\omega t)$ this corresponds to a growth in the oscillation amplitude, *i.e.*, to instability of the mode.

The physical nature of this instability is very simple and closely related to the operation of a normal heat energy: at compression the gas is hotter than normal and this, together with the increased density, causes an increase in the release of energy; this increases the tendency of the gas to expand back towards equilibrium; at expansion the gas is cooler and less dense and hence the energy production is low; this similarly increases the tendency of collapse towards the equilibrium; both effects increase the oscillation amplitude. This mechanism is closely analogous to the operation of a normal car engine where energy is also released (through the ignition of the air–fuel mixture) at the point of maximum compression.

For acoustic modes, which have large amplitudes in the outer part of the star, the damping and excitation are normally dominated by the effects of the flux. This is more complicated and will be discussed in Section 10.2. However, the destabilization through nuclear reactions may play an important role for g modes in several cases; this includes the Sun which becomes unstable towards a few low-order g modes in relatively early phases of its evolution (see, for example, Christensen-Dalsgaard, Dilke & Gough 1974).

10.1.3 Radiative damping of acoustic modes

I now consider the effects on high-order or high-degree acoustic modes and hence neglect the effect of nuclear reactions. As in Problem 2.2(vi) (*cf.* Appendix C) I assume that the flux perturbation is dominated by the term in $\nabla T'$ in equation (10.12), to obtain

$$(\operatorname{div}\mathbf{F})' = \frac{4a\tilde{c}T^4}{3\kappa\rho}|\mathbf{k}|^2\frac{T'}{T} = \omega^2\frac{4a\tilde{c}T^4}{3\kappa\rho c^2}\frac{T'}{T} \,, \tag{10.18}$$

where in the last equality I used the dispersion relation for plane sound waves. Here T'/T can be obtained from the adiabatic relation (10.15) where, in accordance with the treatment of plane sound waves, I neglect derivatives of equilibrium quantities. Hence

$$\frac{T'}{T} = \frac{\delta T}{T} = (\Gamma_3 - 1)\frac{\delta\rho}{\rho} \,; \tag{10.19}$$

As a result equation (10.9) for the damping rate becomes

$$\delta\omega = -\frac{i}{2}\frac{\int_V (\Gamma_3 - 1)^2 \left|\frac{\delta\rho}{\rho}\right|^2 \frac{4a\tilde{c}T^4}{3\kappa\rho c^2}\mathrm{d}V}{\int_V \rho|\boldsymbol{\delta}\mathbf{r}|^2\mathrm{d}V} \, . \tag{10.20}$$

It is evident from equation (10.20) that $\delta\omega$ is negative, *i.e.*, the mode is damped. This is again obvious from physical considerations: the effect of the term in the temperature gradient is to increase the heat flux from regions that are compressed and heated, and to decrease it from regions that are expanded; hence effectively there is heat loss at compression and heat gain at expansion, and this works to dampen the oscillation. It should be pointed out here that the opacity fluctuations, acting through the first term in equation (10.12), may counteract that: if opacity is increased at compression the flux of radiation going out through the star is preferentially absorbed at compression, hence heating the gas and contributing to the excitation of the oscillation. This mechanism, the so-called *Eddington valve*, is responsible for the pulsations of the stars in the instability strip.

To compare with the asymptotic expression derived below it is instructive to write equation (10.20) in terms of $\boldsymbol{\delta}\mathbf{r}$; from the continuity equation we have, still assuming a plane sound wave and using the dispersion relation, that

$$\left|\frac{\delta\rho}{\rho}\right| = |\mathrm{div}\, \boldsymbol{\delta}\mathbf{r}| = |\mathbf{k}||\boldsymbol{\delta}\mathbf{r}| = \frac{\omega}{c}|\boldsymbol{\delta}\mathbf{r}| \, . \tag{10.21}$$

Hence we obtain

$$\frac{\delta\omega}{\omega} = -\frac{i\omega}{2}\frac{\int_V \frac{(\gamma-1)^2}{\gamma^2}\frac{4a\tilde{c}T^4}{3\kappa p^2}|\boldsymbol{\delta}\mathbf{r}|^2\rho\mathrm{d}V}{\int_V \rho|\boldsymbol{\delta}\mathbf{r}|^2\mathrm{d}V} \, , \tag{10.22}$$

by using $c^2 = \gamma p/\rho$; for simplicity I assume that $\Gamma_3 = \Gamma_1 = \gamma$.

It is of some interest to consider also the damping from the asymptotic point of view. I start from the modified dispersion relation derived in Problem 2.2

$$\omega^2 = c^2|\mathbf{k}|^2\phi_\mathrm{F} \, , \tag{10.23}$$

where

$$\phi_\mathrm{F} = \frac{1 + \dfrac{i}{\omega\gamma\tau_\mathrm{F}}}{1 + \dfrac{i}{\omega\tau_\mathrm{F}}} \, , \qquad \tau_\mathrm{F} = \frac{3\kappa\rho p}{4a\tilde{c}(\gamma-1)T^4|\mathbf{k}|^2} = \frac{3\kappa\gamma p^2}{4a\tilde{c}(\gamma-1)T^4\omega^2} \, . \tag{10.24}$$

If nonadiabatic effects are weak, *i.e.*, $\omega\tau_\mathrm{F} \gg 1$, we can write equation (10.23) as

$$\omega^2 = c^2|\mathbf{k}|^2 - c^2|\mathbf{k}|^2(1-\phi_\mathrm{F}) \simeq c^2|\mathbf{k}|^2 - i\omega_\mathrm{ad}\frac{\gamma-1}{\gamma\tau_\mathrm{F}} \, , \tag{10.25}$$

where ω_ad is the frequency in the adiabatic case. Equation (10.25) is a perturbed version of the sound-wave dispersion relation, of the form considered in Appendix B. Hence the effect of the damping on the frequencies can be obtained from equations (B.6) and (B.7) as

$$S\frac{\delta\omega}{\omega} \simeq -\frac{i}{2\omega}\int_{r_\mathrm{t}}^R \left(1 - \frac{L^2c^2}{\omega^2r^2}\right)^{-1/2} \frac{\gamma-1}{\gamma\tau_\mathrm{F}}\frac{\mathrm{d}r}{c} \, , \tag{10.26}$$

where

$$S = \int_{r_t}^{R} \left(1 - \frac{L^2 c^2}{\omega^2 r^2}\right)^{-1/2} \frac{\mathrm{d}r}{c} - \pi \frac{\mathrm{d}\alpha}{\mathrm{d}\omega} \; . \tag{10.27}$$

By substituting the expression for τ_F we finally obtain

$$S \frac{\delta\omega}{\omega} \simeq -\frac{i\omega}{2} \int_{r_t}^{R} \frac{(\gamma-1)^2}{\gamma^2} \frac{4a\tilde{c}T^4}{3\kappa p^2} \left(1 - \frac{L^2 c^2}{\omega^2 r^2}\right)^{-1/2} \frac{\mathrm{d}r}{c} \; . \tag{10.28}$$

This equation essentially corresponds to equation (10.22) if we note that asymptotically $\rho|\boldsymbol{\delta}\mathbf{r}|^2 \mathrm{d}V$ can be identified with $c^{-1}(1 - L^2c^2/\omega^2r^2)^{-1/2}\mathrm{d}r$, to within a constant factor.

10.2 The condition for instability

The arguments presented in this section were originally derived by J. P. Cox. They provide insight into the reason why unstable stars tend to be found in well-defined regions of the HR diagram, particularly the Cepheid instability strip, and are presented here essentially in the form given by Cox (1967, 1974).

Expressing the frequency in terms of real and imaginary parts as $\omega = \omega_r + i\eta$, equation (10.9) can be written approximately as

$$\eta \simeq \frac{C_r}{2\omega_r^2 I} \; , \tag{10.29}$$

where

$$C_r = \mathrm{Re}\left[\int_V \frac{\delta\rho^*}{\rho} (\Gamma_3 - 1)(\rho\epsilon - \mathrm{div}\,\mathbf{F})' \mathrm{d}V\right] \; , \tag{10.30}$$

and

$$I = \int_V \rho |\boldsymbol{\delta}\mathbf{r}|^2 \mathrm{d}V \; . \tag{10.31}$$

Clearly the question of stability or instability depends on the sign of C_r: if $C_r > 0$ the mode is unstable, whereas if $C_r < 0$ the mode is stable.

I consider just the outer parts of the star, where the nuclear energy generation can be neglected. The analysis is restricted to radial oscillations; however, as we know that the behaviour of the modes is largely independent of degree near the surface the results are likely to be at least qualitatively valid for nonradial oscillations as well. Also, I neglect convection. Finally I assume that the oscillations are either quasi-adiabatic or strongly nonadiabatic. In the former region all perturbation quantities can be taken to be real; the strongly nonadiabatic situation is discussed below. Then we can approximate C_r by

$$C_r \simeq -L \int_M \frac{\delta\rho}{\rho} (\Gamma_3 - 1) \frac{\mathrm{d}}{\mathrm{d}m} \left(\frac{\delta L}{L}\right) \mathrm{d}m \; . \tag{10.32}$$

I now assume that $\delta\rho > 0$ everywhere in the region of interest. This would in general hold for the fundamental mode. However, even for higher-order modes the dominant excitation and damping generally take place so close to the surface that $\delta\rho$ has constant sign in this region. It now follows from equation (10.32) that the contribution of a given layer to the damping or excitation depends on the rate of change of δL: if δL increases towards the surface, the layer gives a negative contribution to C_r and hence contributes to the damping, whereas if δL decreases towards the surface, the layer contributes to the excitation. This is entirely consistent with the simple heat-engine argument given in Section 10.1.2, if we notice that we are considering the situation at positive $\delta\rho$, *i.e.*, at compression: if δL increases outwards, more energy leaves the layer at the top than flows in at the bottom; hence there is a net energy loss from the layer at compression, which acts to damp the motion. The reverse is true, of course, if δL decreases towards the surface: then energy is dammed up at compression, and the motion is excited. Clearly, the behaviour of the mode depends on the global effect as determined by the integral in equation (10.32).

We now need to consider the behaviour of the luminosity perturbation in more detail. It is given by an expression corresponding to equation (10.12) for the perturbation in the flux. The radiative luminosity may be expressed as

$$L = -\frac{4a\tilde{c}}{3\kappa} 16\pi^2 r^4 T^4 \frac{\mathrm{d}\ln T}{\mathrm{d}m} \; ; \tag{10.33}$$

hence, expressing the equation in terms of the Lagrangian luminosity perturbation,

$$\frac{\delta L}{L} = 4\frac{\delta r}{r} + (4 - \kappa_T)\frac{\delta T}{T} - \kappa_\rho \frac{\delta\rho}{\rho} - \left(\frac{\mathrm{d}\ln T}{\mathrm{d}m}\right)^{-1} \frac{\mathrm{d}}{\mathrm{d}m}\left(\frac{\delta T}{T}\right) \, . \tag{10.34}$$

For low-order modes one can probably neglect the term in the $\mathrm{d}(\delta T/T)/\mathrm{d}m$, as well as a term in the displacement. Thus we obtain

$$\frac{\delta L}{L} \simeq (4 - \kappa_T)\frac{\delta T}{T} - \kappa_\rho \frac{\delta\rho}{\rho} \, . \tag{10.35}$$

In the region where the oscillations are nearly adiabatic, $\delta T/T \simeq (\Gamma_3 - 1)\delta\rho/\rho$, and hence

$$\frac{\delta L}{L} \simeq \left(\frac{\delta L}{L}\right)_{\mathrm{a}} = \left[(4 - \kappa_T)(\Gamma_3 - 1) - \kappa_\rho\right] \frac{\delta\rho}{\rho} \, . \tag{10.36}$$

In most of the star, κ_ρ is close to unity, κ_T is negative (as, for example, for Kramers opacity) and $\Gamma_3 \simeq 5/3$. Also, $\delta\rho/\rho$ generally increases outwards. Hence it follows from equation (10.36) that in most cases δL increases towards the surface, so that the tendency is towards stability. This is quite reassuring: after all most stars do not show obvious variability, suggesting that special circumstances are required to excite modes to large amplitudes.

In fact, it is clear that there are two circumstances that may give rise to a decrease in $(\delta L)_{\mathrm{a}}$: a strong decrease in Γ_3 or a strong increase in κ_T. Both effects are likely to occur in ionization zones of abundant elements. As an example, Figure 10.1 shows $\Gamma_3 - 1$ in the ionization zone of He in a stellar envelope model (see also Figure 7.15 for the qualitatively very similar behaviour of Γ_1 in the Sun). This occurs because the degree of ionization increases at compression, absorbing the energy that would otherwise have gone towards increasing the temperature and hence reducing the temperature increase. Similarly, although perhaps less obvious, there is a tendency for ionization zones to be associated with 'bumps' in κ_T: it should be noted that since what matters in equations (10.32) and (10.36) is effectively the second derivative of opacity even quite minor features in the opacity can lead to substantial contributions to the excitation, provided that they are confined to a narrow temperature interval. These two mechanisms are generally known as the γ- and κ-mechanisms for mode excitation.

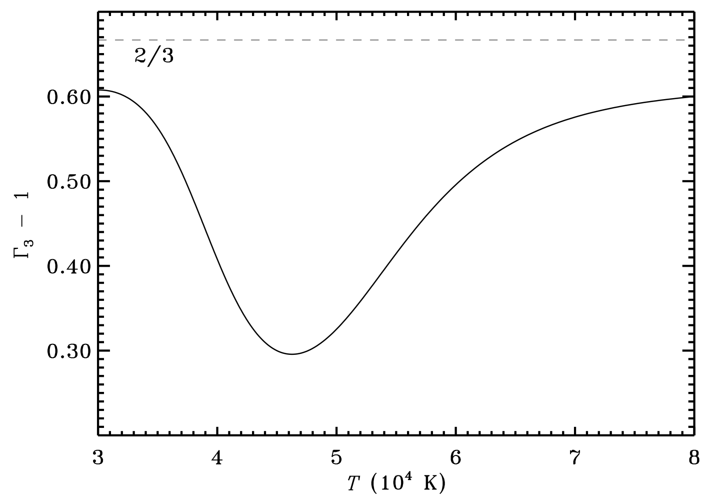

Figure 10.1: $\Gamma_3 - 1$ against temperature in the region of He^+ ionization in an equilibrium model of a stellar envelope.

The description given so far suffers from two problems. First, it is clearly only the lower part of the Γ_3-decrease that will contribute to driving; the upper part similarly contributes to damping, and since $\delta\rho/\rho$ is assumed to increase outwards the damping part is likely to dominate. A similar remark can be made about effects of opacity bumps. Secondly, the argument depends on the quasi-adiabatic approximation, in that the adiabatic relation was used to derive equation (10.35) for δL. The great beauty of Cox's analysis is that it is precisely the transition to nonadiabaticity which is decisive for the occurrence of instability of a star.

To make plausible the transition from adiabaticity to nonadiabaticity I use an argument very similar to the one presented in Section 3.1.4. I write the perturbed energy equation, neglecting the term in ϵ, as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\delta T}{T}\right) - (\Gamma_3 - 1)\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\delta\rho}{\rho}\right) \simeq -\frac{L}{4\pi\rho r^2 c_V T}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\delta L}{L}\right) . \tag{10.37}$$

This can also be written, approximately, as

$$\Delta\left(\frac{\delta L}{L}\right) \sim \Psi\left[\frac{\delta T}{T} - (\Gamma_3 - 1)\frac{\delta\rho}{\rho}\right] , \tag{10.38}$$

where

$$\Psi = \frac{\langle c_V T\rangle \Delta m}{\Pi L} . \tag{10.39}$$

Here $\Delta(\delta L/L)$ is the change in $\delta L/L$ between the surface and the point considered, Δm is the mass outside this point, and $\langle c_V T\rangle$ is a suitable average over this region; also Π is the pulsation period. Thus Ψ has a very simple physical meaning: it is the ratio between the thermal energy stored in the layer outside the point considered and the energy radiated by the star in a pulsation period.

Now equation (10.38) can be understood in simple physical terms. Very near the surface $\Psi \ll 1$: the energy content in the stellar matter is so small that it cannot appreciably affect the luminosity perturbation; thus the luminosity perturbation is *frozen in*, *i.e.*, constant. This is clearly the strongly nonadiabatic limit. Conversely, at great depth $\Psi \gg 1$: the energy content is so large that the flow of energy over a pulsation period has no effect on the energy content; this corresponds to the almost adiabatic case. Thus the transition from adiabatic to nonadiabatic oscillations occurs in the *transition region*, where

$$\frac{\langle c_V T \rangle_{\rm TR} (\Delta m)_{\rm TR}}{\Pi L} \sim 1 \; . \qquad (10.40)$$

The question of stability or instability is now decided by the relative location of the transition region and the relevant ionization zone. It has been shown by Cox that the Cepheid instability strip is controlled by the ionization of He^+; thus in the following I consider only this zone. Also, to understand the location of the instability strip it is convenient to think in terms of varying the radius, and hence the effective temperature, at fixed luminosity.

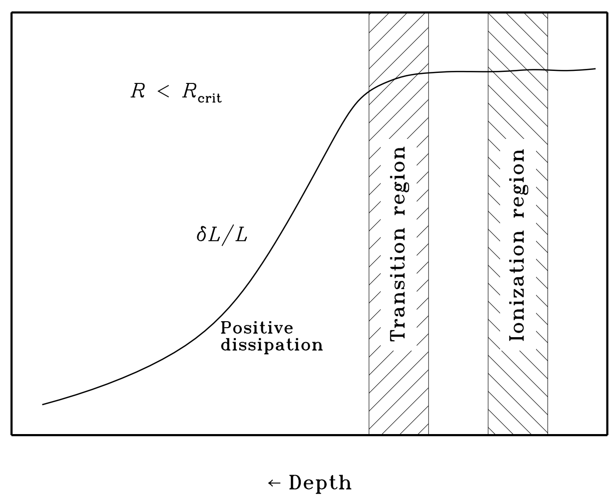

Figure 10.2: $\delta L/L$ at instant of minimum stellar radius and hence maximum compression against depth below the surface (schematic) for a star with $R < R_{\rm crit}$ (see text for explanation of symbols). Only the He^+ ionization zone is shown (after Cox 1967).

Consider first a star of small radius and hence large effective temperature. Here the He^+ ionization zone lies close to the surface, *i.e.*, very likely above the transition region (*cf.* Figure 10.2). Below the transition region $\delta L/L$ follows the adiabatic behaviour and hence increases outwards; this contributes to the damping. Above the transition region δL is approximately constant, and there is no contribution to the excitation and damping. Thus the net effect is that $C_{\rm r} < 0$, *i.e.*, the star is stable.

Now increase the radius, and hence reduce $T_{\rm eff}$, sufficiently that the transition region coincides with the He^+ ionization zone. As illustrated in Figure 10.3, at this critical radius

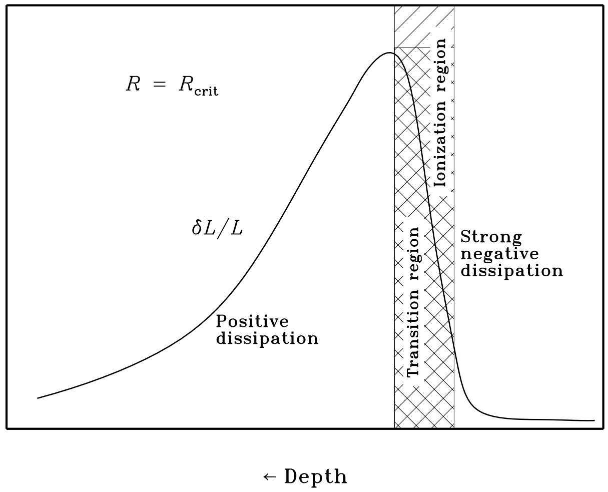

Figure 10.3: $\delta L/L$ at instant of minimum stellar radius and hence maximum compression against depth below the surface (schematic) for a star with $R = R_{\rm crit}$ (see text for explanation of symbols). Only the He^+ ionization zone is shown (after Cox 1967).

$R_{\rm crit}$ the situation changes dramatically. We still get damping in the interior of the star; however, the lower part of the ionization zone now contributes strongly to the excitation, and the corresponding damping in the upper part of the ionization zone is absent because the luminosity perturbation is frozen in here. Thus in this case there is chance for instability. This is precisely what happens: the point where $R = R_{\rm crit}$ corresponds to the location of the instability strip.

Finally, at even larger radius and lower $T_{\rm eff}$ the entire ionization zone lies in the quasi-adiabatic region and hence it makes both positive and negative contributions to the excitation. As argued above, the general increase towards the surface of $\delta\rho/\rho$ makes it plausible that the net effect is damping of the modes. In fact, computations show that it is difficult to reproduce the lower (so-called red) edge of the instability strip unless effects of perturbations in the convective flux are taken into account.

This argument may be more quantitative, to determine the approximate location of the instability strip. In fact, it is not difficult to obtain a relation that determines the slope of the strip (see Problem 6.2 in Appendix C). It was arguments of this kind which first led Cox to identify the He^+ ionization as being mainly responsible for the Cepheid instability.

The location of the transition region, as given in equation (10.40), depends on the period of oscillation. I have so far argued for the behaviour of a single mode (although the changing radius would also tend to increase the period and hence push the transition region deeper). However, it should be noted that higher-order modes would tend to have transition regions closer to the surface. It follows that they should become unstable at higher effective temperatures. This is indeed confirmed by more detailed stability calculations.

The arguments as given here refer specifically to the Cepheid instability strip. However, very similar arguments can be applied to other driving mechanisms, at least in fairly hot stars where convection can be neglected. Thus any suitable feature that may cause a substantial dip in $(\delta L/L)_{\rm a}$ might be expected to give rise to an instability region. It has been found, through improvements in the treatment of iron line contributions, that there is a bump in the opacities near temperatures of 2×10^5 K which can account for the β Cephei and other B star pulsations in this way (*e.g.* Moskalik & Dziembowski 1992); before these improvements the origin of B-star pulsations was a major mystery. A similar mechanism is responsible for the excitation of g modes in at least some pulsating white dwarfs (*e.g.* Winget *et al.* 1982).

10.3 Stochastic excitation of oscillations

Nonadiabatic calculations taking convection into account generally find that modes in stars on the cool side of the instability strip are stable. In particular, this is the case for the modes observed in the Sun (*e.g.* Balmforth 1992a). Thus the presence of oscillations in the Sun and other cool stars requires other excitation mechanisms. In these stars the convective motion near the surface likely reaches speeds close to that of sound. Such turbulent motion with near-sonic speed is an efficient source of acoustic radiation, and it is likely that this 'noise' excites the normal modes of the star, to the observed amplitude.

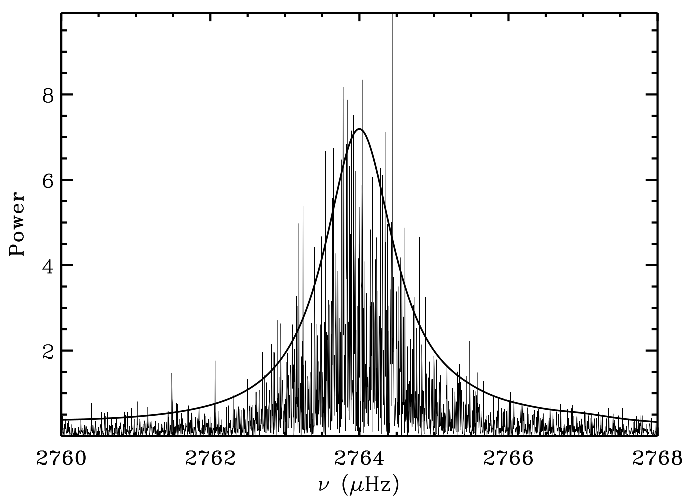

Figure 10.4: Observed spectrum, from Doppler observations with the BiSON network, of a single radial mode of solar oscillations. The smooth curve shows the fitted Lorentzian profile, multiplied by three for clarity. (See Chaplin *et al.* 2002.)

Since the excitation is caused by a very large number of convective elements, the driving is essentially random. The problem of a damped oscillator driven by random forcing was considered by Batchelor (1956), and the analysis is discussed in Problem 6.1 in Appendix C. The outcome is that the average power spectrum resulting for a mode of frequency ω_0, and damping rate η, is approximately

$$\langle P(\omega)\rangle \simeq \frac{1}{4\omega_0^2}\frac{\langle P_f(\omega)\rangle}{(\omega-\omega_0)^2+\eta^2} , \qquad (10.41)$$

where $\langle P_f(\omega)\rangle$ is the average power spectrum of the forcing function. If the forcing spectrum is a slowly varying functions of frequency, the result is therefore a Lorentzian spectrum, with a width determined by the linear damping rate of the mode.

If a single realization, rather than the average, of the spectrum is considered, as is generally the case for observations of stellar oscillations, the result is a random function with a Lorentzian envelope. An example is shown in Figure 10.4, based on observations of solar oscillations with the BiSON network. Such Lorentzian profiles are often assumed in the fits carried out to determine the frequency and other properties of the modes. It should be noticed, however, that the observed profiles show definite asymmetries and hence cannot be strictly represented by Lorentzian profiles. This behaviour can be understood from the detailed properties of the excitation, in particular the fact that the dominant contributions to the forcing arise from a relatively thin region (*e.g.*, Duvall *et al.* 1993; Gabriel 1993, 2000; Roxburgh & Vorontsov 1995; Abrams & Kumar 1996; Nigam & Kosovichev 1998; Rast & Bogdan 1998; Rosenthal 1998). Neglecting this effect in the fitting causes systematic errors in the inferred frequencies; however, it appears that these are of a form similar to the effects of the near-surface errors [*i.e.*, the term $Q_{nl}^{-1}\mathcal{G}(\omega_{nl})$ in equation (9.32)], and hence have no effect on the results of structure inversion (*e.g.* Rabello-Soares *et al.* 1999b; Basu *et al.* 2000). Observational determination of the asymmetry does, however, provide constraints on the properties of subsurface convection (Chaplin & Appourchaux 1999; Kumar & Basu 1999; Nigam & Kosovichev 1999).

As a result of the stochastic nature of the excitation, the observed amplitude of a mode varies over time. The statistical properties of this variation were discussed by Kumar, Franklin & Goldreich (1988) and Chang & Gough (1998). If the modes are observed over a time short compared with the damping time, the energy distribution is exponential,

$$p(E)\mathrm{d}E = \langle E\rangle^{-1}\exp(-E/\langle E\rangle)\mathrm{d}E , \qquad (10.42)$$

where $\langle E\rangle$ is the average energy, and the energy E is proportional to the squared amplitude.

It is straightforward, and instructive, to simulate such stochastically excited, damped, oscillations. An example of such a simulation, for a long-period variable, is illustrated in Figure 10.5. It is evident that the amplitude varies strongly and in an irregular fashion, and hence at any given time there is a significant probability that any given mode may be invisible; this must be kept in mind in the interpretation of such pulsating stars. Panel b) shows the distribution of mode energy, obtained by analyzing the time series in 1-year segments. Here N is the total number of segments, and n is a scaled binned number of realizations,

$$n = \frac{\Delta n(E)}{\exp(\Delta E/2\langle E\rangle) - \exp(-\Delta E/2\langle E\rangle)} , \qquad (10.43)$$

where $\Delta n(E)$ is the number of realizations in the interval $[E-\Delta E/2, E+\Delta E/2]$. It may be shown that n/N behaves like $\exp(-E/\langle E\rangle)$ (*cf.* Chang & Gough 1998); as is clear from Figure 10.5b the simulated data do indeed have this property. Very interestingly, the observed distribution of solar oscillation amplitudes satisfies this relation quite closely (*e.g.* Chaplin *et al.* 1997). An example, based on BiSON data, is shown in Figure 10.6.

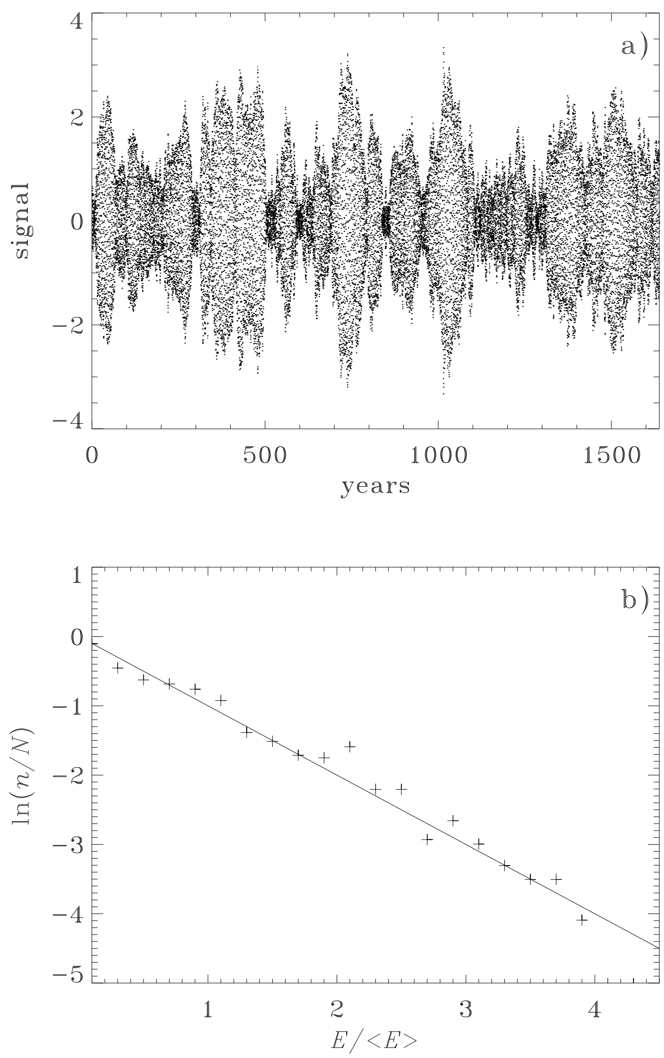

Figure 10.5: Artificial time series for an oscillation with a period of 82 days, a damping time of 60 years and a sampling-time interval of 20 days. The top panel shows the computed time series which, as indicated, covers about 1600 years. In the bottom panel the points show the binned, normalized distribution of mode power, in units of the mean power; the line corresponds to the expected exponential distribution in equation (10.42) (see text). (From Christensen-Dalsgaard *et al.* 2001.)

Exercise 10.2:

Verify this property of the distribution, as described by equation (10.43).

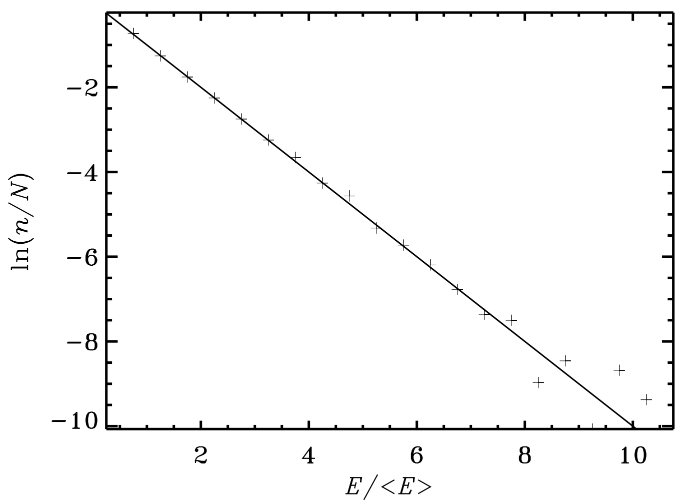

Figure 10.6: Binned, normalized distribution of observed solar mode power, in units of mean power; this is based on 3368 individual samples, each containing 14 modes, of BiSON observations. The line corresponds to the expected exponential distribution in equation (10.42). (See Chaplin *et al.* 1997.)

The distribution function in equation (10.42) also defines the relation between the average $\langle A \rangle$ and the standard deviation $\sigma(A)$ of the amplitude:

$$\sigma(A) = \left(\frac{4}{\pi} - 1\right)^{1/2} \langle A \rangle \simeq 0.52 \langle A \rangle \; . \tag{10.44}$$

It was noticed by Christensen-Dalsgaard, Kjeldsen & Mattei (2001) that observed amplitudes of semiregular variables on the red-giant branch approximately followed this relation, suggesting that their variability may have a cause similar to the solar oscillations.

As indicated by equation (10.41) this excitation mechanism results in a definite prediction of the oscillation amplitude, given a model for the power in the stochastic forcing. This can be evaluated from models of convection, such as the mixing-length description. A rough estimate was made by Christensen-Dalsgaard & Frandsen (1983a); following Goldreich & Keeley (1977) they assumed equipartition between the energy in a single mode of oscillation and the energy of a convective eddy with a time scale corresponding to the period of the mode. The results were analyzed by Kjeldsen & Bedding (1995) who found, as already discussed in Section 2.4.1, that the amplitudes scaled as L/M (*cf.* eq. 2.44). A more careful calculation was carried out by Houdek *et al.* (1999), who determined the damping or excitation rates of radial modes, using a nonlocal mixing-length description of the interaction between convection and pulsation; for the stable modes they estimated the stochastically excited amplitudes, from the computed damping rates and a mixing-length calculation of the energy input to the modes from convection. The results are summarized in Figure 10.7. It should also be noted that computations by Stein & Nordlund (2001) of the energy input from convection to the oscillations, based on detailed hydrodynamical

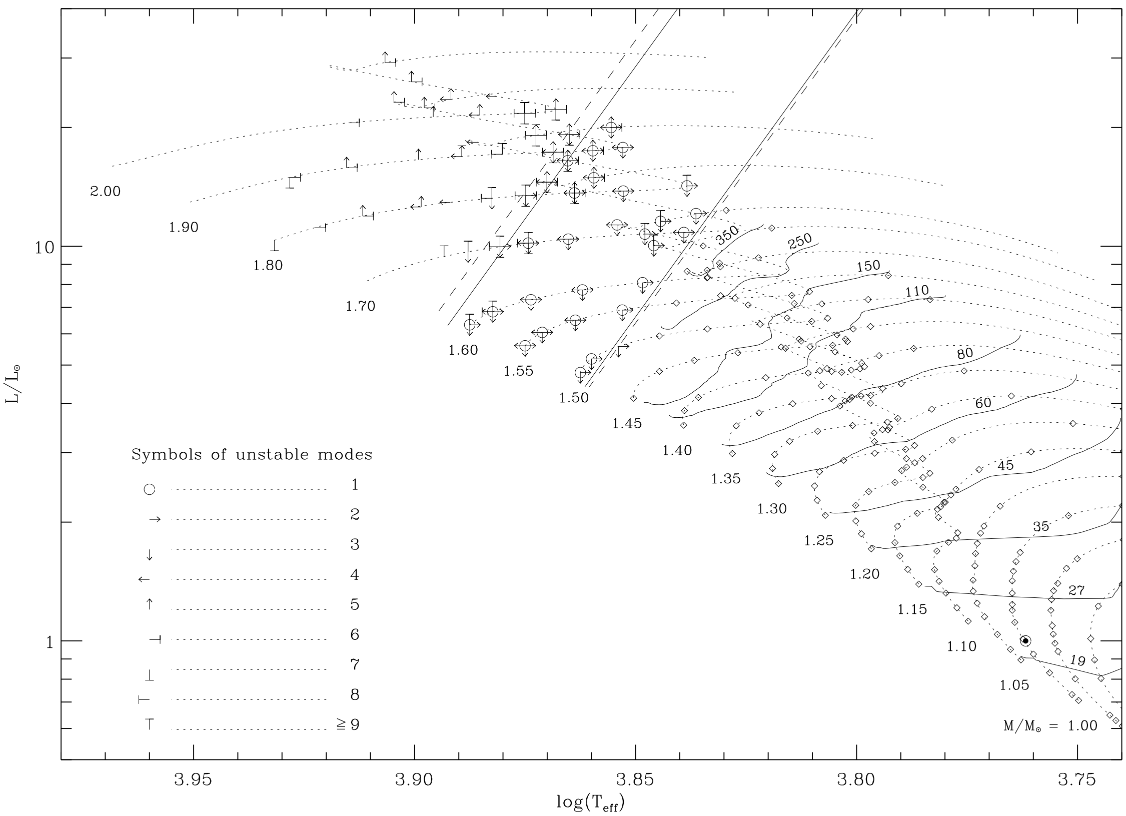

Figure 10.7: Unstable modes and mean velocity amplitudes of stochastically excited modes, for radial oscillations. Evolution tracks, at the masses indicated, are shown with dotted curves, some models being marked with diamonds. Selected models with unstable modes are indicated by the symbols, as listed in the figure; note that, as argued in Section 10.2, the higher-order modes tend to be excited in models with higher effective temperature. The solid and dashed straight lines indicate the instability strips of the $n = 1$ and 2 modes, respectively. The contours to the right of the instability strip show computed velocity amplitudes, averaged over frequency; the values of the amplitudes, in $\mathrm{cm\,s^{-1}}$, are given. For the Sun, indicated by $\odot$, the predicted mean amplitude is $20\,\mathrm{cm\,s^{-1}}$. (From Houdek *et al.* 1999.)

simulations, have yielded results in general agreement with the observed properties of solar oscillations.

The stochastic mechanism is expected to result in the excitation of all modes in a broad range of frequencies, with amplitudes reflecting the presumed slow frequency dependence of the forcing function. This property is indeed observed in the Sun and in the few cases where solar-like oscillations have been observed in other stars (see Section 2.4.1). It greatly simplifies the identification of the modes, compared with oscillations excited through radiative instability. In the latter case the mechanisms determining the final amplitude, and hence the selection of modes which reach an observable level, are unknown and apparently lead to detectability, with current techniques, of only a fairly small subset of the unstable modes.

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